



Pencil and Paper Procedures September 2021



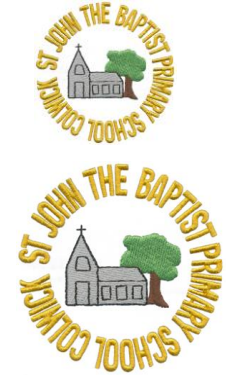
Life in all its fullness

An exciting, quality environment



St John the Baptist C of E Primary School

Pencil and paper procedures



Background to the policy

This policy contains the key pencil and paper procedures that will be taught within our school. It has been written to ensure consistency and progression throughout the school and reflects a whole school agreement.

Although the focus of the policy is on pencil and paper procedures it is important to recognise that the ability to calculate mentally lies at the heart of the Mathematics Curriculum. The mental methods in will be taught systematically from Reception onwards and pupils will be given regular opportunities to develop the necessary skills. However mental calculation is not at the exclusion of written recording and should be seen as complementary to and not as separate from it. In every written method there is an element of mental processing. Sharing written methods with the teacher encourages children to think about the mental strategies that underpin them and to develop new ideas. Therefore written recording both helps children to clarify their thinking and supports and extends the development of more fluent and sophisticated mental strategies.

During their time at this school children will be encouraged to see mathematics as both a written and spoken language. Teachers will support and guide children through the following important stages:

- developing the use of pictures and a mixture of words and symbols to represent numerical activities;
- using standard symbols and conventions;
- use of jottings to aid a mental strategy;
- use of pencil and paper procedures;
- use of a calculator;
- use of specific maths manipulatives such as Numicon and Demines;
- use of ICT e.g. apps and games.

This policy concentrates on the introduction of standard symbols, the use of the empty number line as a jotting to aid mental calculation and on the introduction of pencil and paper procedures. It is important that children do not abandon jottings and mental methods once pencil and paper procedures are introduced. Therefore children will always be encouraged to look at a calculation/problem and then decide which is the best method to choose – pictures, mental calculation with or without jottings, structured recording or a calculator. Our long-term aim is for children to be able to select an efficient method of their choice (whether this be mental, written or in upper Key Stage 2 using a calculator) that is appropriate for a given task. They will do this by always asking themselves:

'Can I do this in my head?'

'Can I do this in my head using drawings or jottings?'

'Do I need to use a pencil and paper procedure?'

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'Do I need a calculator?'

Our curriculum progression map is based on the White Rose scheme which heavily utilises the idea of 'varied fluency'. Varied fluency is an approach to teaching key concepts through a variety of concrete, abstract and pictorial methods whilst incorporating rich mathematical language.

Aims and objectives

We will endeavour to instill the Christian Values of resilience in the teaching of the calculation methods outlined in this policy.

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FOUNDATION STAGE

Addition

Use of pictures/marks to support learning:

$2 + 2 = 4$

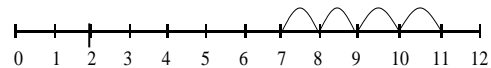
$2 + 1 = 3$

$2 + \square = \square$

$2 + 1 = 3$

Use of number lines:

(Counting on)
e.g. $7 + 4 = 11$

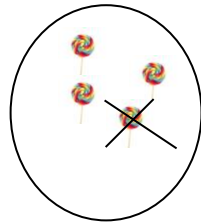


Recording by - drawing jumps on prepared lines

Subtraction

Use of pictures/marks to support learning:

$4 - 1 = 3$

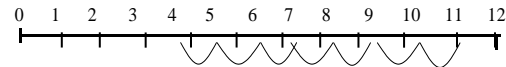


Record the number sentence alongside diagram

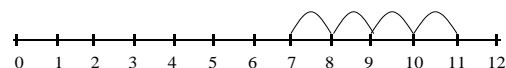
Use of number lines (numbered):

$11 - 7$

(Counting back)



The difference between 7 and 11
(Counting up)

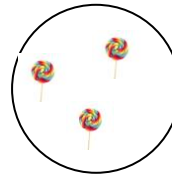


Multiplication

Use vocabulary
'groups'
'lots of'

Use of pictures/marks to support learning:

2 groups of 3
(Showing number sentence 4×2)



Division

Use of vocabulary 'sharing'

Use of pictures/marks to support learning:

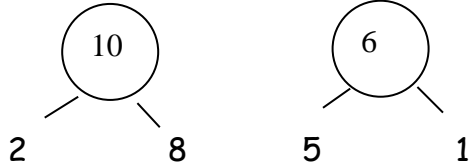
4 shared between 2
(Showing number sentence $4 \div 2$)



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Part Whole Model



Bar models

Bead strings

$\text{○○○○○} - \text{○} = 6$
 $5 + 1 = 6$

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YEAR 1

Addition

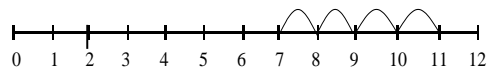
Use of pictures/marks to support learning as in Foundation Stage.

Egg Method (higher ability need partitioning in place):

$$\begin{array}{ccc} 10 & 10 & 20 \\ \textcircled{16} & + & \textcircled{12} = \textcircled{28} \\ 6 & 2 & 8 \end{array}$$

Number lines (numbered):

$$7 + 4 = 11$$



Recording by - drawing jumps on prepared lines

+ = signs and missing numbers:

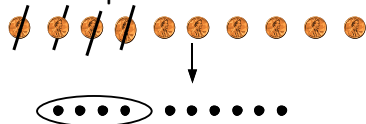
$$\begin{array}{ll} 3 + 4 = \square & \square = 3 + 4 \\ 3 + \square = 7 & 7 = \square + 4 \\ \square + 4 = 7 & 7 = 3 + \square \\ \square + \nabla = 7 & 7 = \square + \nabla \end{array}$$

Promoting covering up of operations and numbers.

Subtraction

Pictures / marks:

Webster spent 4p. What was his change from 10p?



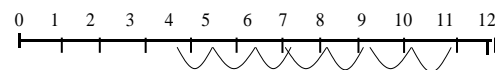
- = signs and missing numbers:

$$\begin{array}{ll} 7 - 3 = \square & \square = 7 - 3 \\ 7 - \square = 4 & 4 = \square - 3 \\ \square - 3 = 4 & 4 = 7 - \square \\ \square - \nabla = 4 & 4 = \square - \nabla \end{array}$$

Number lines (numbered):

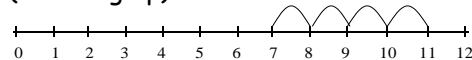
$$11 - 7$$

(Counting back)



The difference between 7 and 11

(Counting up)

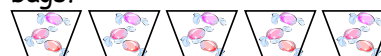


Recording by - drawing jumps on prepared lines

Multiplication

Pictures and symbols

There are 3 sweets in one bag.
How many sweets are there in 5 bags?



Children to count them all

Drawing groups on a line:

Jumping up in 'groups of'



Using bead strings to support.

Move onto numbered number line

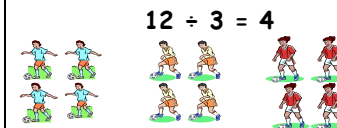
jumping up in 'groups':

Division

Use of vocabulary 'sharing'

Use of pictures/marks to support learning:

12 children are shared into teams of 4 to play a game. How many teams are there?

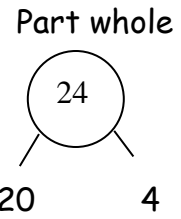


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Add column method with partition

e.g
$$\begin{array}{r} 18 \\ + 14 \\ \hline 32 \\ 1 \end{array}$$



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YEAR 2

Addition

+ = signs and missing numbers:

Continue using a range of equations as in Year 1 but with appropriate, larger numbers. Extend to

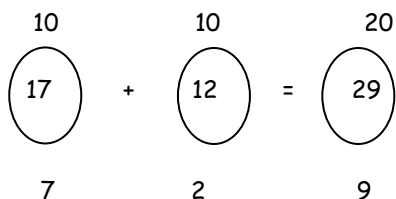
$14 + 5 = 10 + \square$

and adding three numbers

$32 + \square + \square = 100$ $35 = 1 + \square + 5$

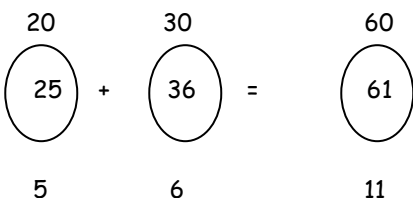
Egg Method (need partitioning in place):

$17 + 12 = 29$



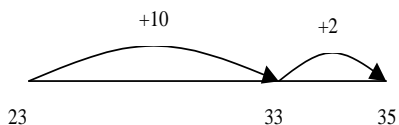
Move onto units crossing the tens boundary:

$25 + 36 = 61$



Number lines using partitioning knowledge:

$23 + 12 = 35$



Recording by - drawing jumps on prepared lines (higher ability draw own)

Subtraction

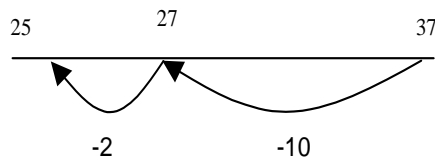
- = signs and missing numbers:

Continue using a range of equations as in Year 1 but with appropriate numbers. Extend to $14 + 5 = 20 - \square$

Number lines:

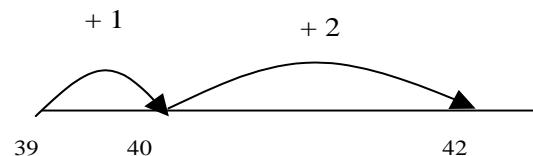
(Counting back)

$37 - 12 = 25$



Finding the difference between 39 and 42 (Counting up)

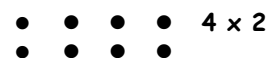
$42 - 39 = 3$



Multiplication

Arrays and repeated addition:

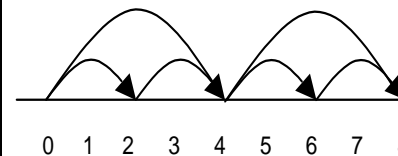
Read as rows - 2 lots of 4



Number lines when multiplication is repeated addition:

4×2

..... is $2 + 2 + 2 + 2$



Division

Understand division as sharing and grouping

Use of pictures/marks to support learning:

$6 \div 2$

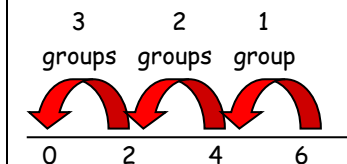
Sharing - 6 sweets are shared between 2 people. How many do they have each?



Use of number line for grouping:

$6 \div 2$

Grouping - There are 6 sweets. How many people can have 2 each? (How many groups of 2 make 6?)



"How many groups of 2 can you make from 6?"

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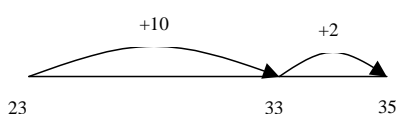
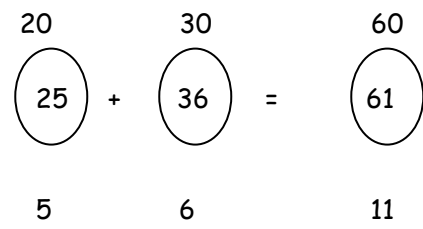
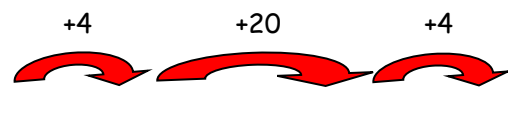
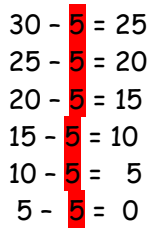
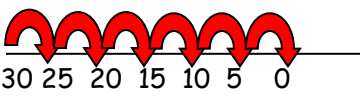


<p><u>Addition</u></p> <p>2 digit + 1 digit column method</p> <p>e.g. 12</p> $\begin{array}{r} + 4 \\ \hline 16 \end{array}$ <p>2 digit + 2 digit</p> <p>e.g. 42</p> $\begin{array}{r} + 33 \\ \hline 75 \end{array}$	<p>2 digit +2 digit (crossing 10s)</p> <p>move on to 47</p> $\begin{array}{r} + 18 \\ \hline 65 \\ 1 \end{array}$	<p><u>Subtraction</u></p> <p>2 digit - 1 digit column method</p> <p>e.g. 48</p> $\begin{array}{r} - 5 \\ \hline 43 \end{array}$ <p>2 digit - 2 digit</p> <p>76</p> $\begin{array}{r} - 32 \\ \hline 44 \end{array}$	<p>2 digit - digit (with borrowing and exchanging)</p> <p>move on to ²3¹4</p> $\begin{array}{r} - 17 \\ \hline 17 \end{array}$
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YEAR 3

Addition	Subtraction	Multiplication	Division									
<p><u>Number line:</u></p> <p>$23 + 12 = 35$</p>  <p>Recording by - drawing jumps on prepared lines (higher ability draw own)</p> <p><u>Egg Method (need partitioning in place):</u></p> <p>crossing the tens boundary:</p> <p>$25 + 36 = 61$</p>  <p><u>Move onto partitioning:</u></p> $\begin{array}{r} 42 + 36 = \\ 40 + 30 = 70 \\ 2 + 6 = + 8 \\ \hline 78 \end{array}$	<p><u>Number line method TU - TU:</u></p> <p>Finding the difference between 84 and 56 (Counting on)</p> <p>$84 - 56 = 28$</p>  <p>56 60 80 84</p> <p>Subtracting by adjusting from a multiple of 10 i.e. $-9 = -10 + 1$ / $-11 = -10 -1$</p> <p><u>Partitioning standard method (TU):</u></p> $\begin{array}{r} 57 - 36 \\ 50 + 7 \\ - 30 + 6 \\ \hline 20 + 1 = 21 \end{array}$ <p><u>Move onto a standard column method once place value is secure (not crossing the ten's boundary):</u></p> $\begin{array}{r} 36 - 24 = 12 \\ 36 \\ - 24 \\ \hline 12 \end{array}$	<p><u>Written Method</u></p> <p>2 digit x 1 digit mental</p> $\begin{array}{r} 23 \\ \times 8 \\ \hline 184 \end{array}$ <p>12</p> <p><u>Grid Method TU x U:</u></p> <p>$23 \times 8 =$</p> <table border="1" data-bbox="1218 747 1575 974"> <tr> <td>2</td> <td>3</td> <td>X</td> </tr> <tr> <td>1</td> <td>2</td> <td>8</td> </tr> <tr> <td>6</td> <td>4</td> <td></td> </tr> </table> <p>1 8 4</p> <p><u>Long multiplication standard method TU by U only:</u></p> $\begin{array}{r} 23 \\ \times 8 \\ \hline 184 \end{array}$ <p>$23 \times 8 = 184$</p> <p>long</p> $\begin{array}{r} 23 \\ \times 8 \\ \hline 184 \end{array}$	2	3	X	1	2	8	6	4		<p><u>Informal diagram:</u></p> <p>$30 \div 5 = 6$</p>  <p>6 groups of 5</p> <p><u>Number line method:</u></p> <p>Used where the number line is getting smaller.</p> <p>$30 \div 5 = 6$</p> 
2	3	X										
1	2	8										
6	4											

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<p><u>Addition</u> 2 digit + 2 digit (with carrying)</p> $\begin{array}{r} 47 \\ +18 \\ \hline 65 \\ 1 \end{array}$ <p>Use the same method for adding:</p> <p>3 digit + 1 digit 236</p> <p>3 digit + 2 digit $\begin{array}{r} +327 \\ \hline \end{array}$</p> <p>3 digit + 3 digit $\begin{array}{r} 563 \\ \hline 1 \end{array}$</p>	<p><u>Subtraction</u> 2 digit - 2 digit (to include borrowing)</p> $\begin{array}{r} 23 \\ -17 \\ \hline 17 \end{array}$ <p>and 3 digit - 3 digit</p> $\begin{array}{r} 556 \\ -327 \\ \hline 236 \end{array}$	<p>Short 23</p> $\begin{array}{r} \times 8 \\ \hline 184 \\ \hline 12 \end{array}$ <p>Application of times tables knowledge (2,5,10) and 3,4, 8 times table</p>	
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YEAR 4

YEAR 4																
Addition	Subtraction	Multiplication	Division													
<p><u>Partitioning expanded column method:</u></p> $42 + 36 =$ $40 + 30 = 70$ $2 + 6 = + 8$ $\underline{\quad 78 \quad}$ with carrying <p>4 digit + 4 digit column method (with carrying)</p> $\begin{array}{r} 3625 \\ + 2517 \\ \hline 6142 \\ 11 \end{array}$	<p><u>Number line HTU - TU</u></p> <p>Finding the difference between 184 and 56 (Counting on)</p> $184 - 56 = 128$ <p>56 60 100 80 84</p> <p><u>Partitioning standard method (HTU):</u></p> $457 - 236$ $400 + 50 + 7$ $- 200 + 30 + 6$ <hr/> $200 + 20 + 1 = 221$ <p><u>Move onto a standard column method once place value is secure (cross the tens):</u></p> $34 - 28 =$ $\begin{array}{r} 2 \\ 3 \cancel{1} 4 \\ - 28 \\ \hline 06 \end{array}$	<p><u>Grid method TU x TU:</u></p> $48 \times 56 =$ If needed <table border="1"> <tr> <td>X</td> <td>50</td> <td>6</td> </tr> <tr> <td>40</td> <td>2000</td> <td>240</td> </tr> <tr> <td>8</td> <td>400</td> <td>48</td> </tr> </table> <p>$2000 + 240 + 400 + 48 = 2688$</p> <p>Application of times tables knowledge up to 12×12</p> <p>2 digit x 1 digit written method</p> $\begin{array}{r} 24 \\ \times 3 \\ \hline 72 \\ 1 \end{array}$ <p>2 digit x 2 digit</p> $\begin{array}{r} 57 \\ \times 62 \\ \hline 3420 \\ 3534 \end{array}$ <p>3 digit x 1 digit</p> $\begin{array}{r} 326 \\ \times 4 \\ \hline 1304 \\ 112 \end{array}$	X	50	6	40	2000	240	8	400	48	<p><u>Key facts box used to support...</u></p> <table border="1"> <tr> <td>5 x</td> </tr> <tr> <td>10 x</td> </tr> <tr> <td>20 x</td> </tr> <tr> <td>100 x</td> </tr> </table> <p><u>Horizontal number line:</u> Used where the number line is getting bigger (when using large numbers).</p> $37 \div 5 = 7 \text{ r}2$ <p>1 2 3 4 5 6 7 (groups)</p> <p>0 5 10 15 20 25 30 35 37</p> <p><u>Chunking number line:</u></p> $37 \div 5 = 7 \text{ r}2$ <p>6 groups 1 group</p> <p>37 7 2</p>	5 x	10 x	20 x	100 x
X	50	6														
40	2000	240														
8	400	48														
5 x																
10 x																
20 x																
100 x																

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Subtraction

4 digit - 4 digit (with borrowing)

$$\begin{array}{r} 561342 \\ -3625 \\ \hline 2517 \end{array}$$

$$-3625$$

$$\hline 2517$$

Long multiplication standard method TU by TU by partitioning:

$$23 \times 86 =$$

$$\begin{array}{r} 23 \\ \times 86 \\ \hline \end{array}$$

$$6 \times 3 = 18$$

$$6 \times 20 = 120$$

$$80 \times 3 = 240$$

$$80 \times 20 = 1600 +$$

$$\hline 1978$$

$$\hline$$

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YEAR 5

Addition	Subtraction	Multiplication	Division															
<p><u>Partitioning expanded column method:</u></p> $\begin{array}{r} 42 + 36 = \\ 40 + 30 = 70 \\ 2 + 6 = + 8 \\ \hline + 78 \\ \hline \end{array}$ <p><u>Formal method.</u></p> <p>Showing numbers carried underneath</p> $\begin{array}{r} 358 \\ + 73 \\ \hline 431 \\ 11 \end{array}$ <p>Extend to numbers with at least four digits</p> $3587 + 675 = 4262$ $\begin{array}{r} 3587 \\ + 675 \\ \hline 4262 \\ 111 \end{array}$ <p>Revert to expanded methods if the children experience any difficulty.</p> <p>Extend to decimals.</p>	<p><u>Finding differences by counting on.</u></p> <p><u>Partitioning standard method supported by number line:</u></p> $4008 - 2995 = 1013$ <p>Moving onto numbers with two decimal places. E.g. £235 + £ 33.75.</p> <p><u>Move onto a standard column method once place value is secure (Th H T U):</u></p> $2184 - 156$ $\begin{array}{r} 7 \\ 2 8 \\ - 1 5 \\ \hline 2 2 \end{array}$ <p>Bigger than 4 digit - 4 digit</p> $\begin{array}{r} 21784 \\ - 1156 \\ \hline 2028 \end{array}$	<p><u>Grid method HTU x TU:</u></p> $372 \times 24 \text{ is approximately } 400 \times 20 = 8000$ <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>x</td> <td>300</td> <td>70</td> <td>2</td> </tr> <tr> <td>20</td> <td>6000</td> <td>1400</td> <td>40</td> </tr> <tr> <td>4</td> <td>1200</td> <td>280</td> <td>8</td> </tr> </table> <p>Extend to decimals with up to two decimal places.</p> <p>Application of times tables knowledge up to 12×12</p> <p><u>Written method</u></p> <p>4 digit x 1 digit</p> $\begin{array}{r} 2341 \\ \times 6 \\ \hline 14046 \end{array}$ <p>3 digit x 2 digit</p> $\begin{array}{r} 278 \\ \times 36 \\ \hline 1722 \\ +8610 \\ \hline 10332 \end{array}$	x	300	70	2	20	6000	1400	40	4	1200	280	8	<p><u>Key facts box used to support...</u></p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>5 x</td> </tr> <tr> <td>10 x</td> </tr> <tr> <td>100 x</td> </tr> </table> <p><u>Vertical chunking number line:</u> (if necessary make the steps smaller)</p> $61 \div 8 =$ <p><u>Chunking method:</u></p> <p>$72 \div 5$ lies between $50 \div 5 = 10$ and $100 \div 5 = 20$</p> $\begin{array}{r} 72 \\ - 50 \quad (10 \text{ groups}) \text{ or } (10 \times 5) \\ \hline 22 \\ - 20 \quad (4 \text{ groups}) \text{ or } (4 \times 5) \\ \hline 2 \end{array}$ <p>Answer : 14 remainder 2</p> <p><u>Bus shelter secured:</u></p> $\begin{array}{r} 325 \text{ r } 2 \\ 3 \overline{) 9717} \end{array}$	5 x	10 x	100 x
x	300	70	2															
20	6000	1400	40															
4	1200	280	8															
5 x																		
10 x																		
100 x																		

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YEAR 6

Addition	Subtraction	Multiplication	Division
<p><u>Partitioning:</u></p> <p>Partition into hundreds, tens, ones and decimal fractions and recombine Either partition both numbers and recombine or partition the second number only e.g.</p> $35.8 + 7.3 = 35.8 + 7 + 0.3$ $= 42.8 + 0.3$ $= 43.1$ <p><u>Standard Column Method:</u> Extend to numbers with any number of digits and decimals with 1 and 2 decimal places. $124.9 + 117.25 = 242.15$</p> $\begin{array}{r} 124.90 \\ + 117.25 \\ \hline 242.15 \\ 11 \end{array}$ <p>Revert to expanded methods if the children experience any difficulty. Extend to decimals (either one or two decimal places).</p>	<p><u>Standard compact method:</u></p> $\begin{array}{r} 39 \\ 410\ 108 \\ - 2995 \\ \hline 1013 \end{array}$ <p>Moving onto decimal numbers</p>	<p><u>Standard Method:</u> $236 \times 23 =$</p> $\begin{array}{r} 236 \\ \times 27 \\ \hline 1652 \\ + 4720 \\ \hline 6372 \end{array}$ <p>Time Tables All up to 12×12</p>	<p><u>Bus shelter secured:</u></p> $\begin{array}{r} 325\ r2 \\ 3 \overline{)9717} \end{array}$

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POLICY REASONING

- The above policy shows the main methods to be employed by class teachers for on track children in each particular year.
- There is scope for class teachers to 'layer' up or back in their method choice depending on children's ability.
- For those children who cannot access the main written method depicted for their year, class teachers should make use of the wide range of alternative written methods available so every child has a successful age appropriate written method to choose from. The policy therefore allows teachers to individualise where necessary.
- Year 4, 5, 6 integers are much larger than those used in Year 3. As a result, although counting back will be done orally, written methods where counting back is used can be unwieldy and inappropriate. Therefore written methods in subtraction and division change from counting back to counting up. It is assumed that the children will understand the concept of the operation as a result of earlier counting back methods employed in Years 3, 2 and 1.

Reviewed: September 2021

Review: September 2023

Reviewed by: P Seaton

Governor: